Trajectories that Fly by Jupiter and Saturn and Return to Earth

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Introduction

THIS Note presents two classes of multiplanet flyby missions that perform free flybys at both Jupiter and Saturn and return to Earth. The energy and the flight times of these trajectories are about the same as the JSUN, JSP, or JUN missions,‡ and many of these trajectories have entry velocities at Earth that are low enough to permit an unmanned probe to return safely. The opportunities for these trajectories occur at approximately 400-day intervals when Jupiter is the first planet and at 378-day intervals when Saturn is first. The period of time investigated is slightly more than one synodic period of Jupiter and Saturn (19.85 years).

Method of Solution

The approach employed in solving the dual-planet flyby problem is that of defining the problem in terms of its fundamental independent and dependent variables (or constraints) and then of solving for the independent variables, using a multivariable search and minimization technique.[1] The independent variables are the departure date from Earth and the dates on which each of the planets in the sequence is encountered. The constraints are that no velocity impulse is required at Jupiter or Saturn; the altitude at Jupiter or Saturn is specified, and the departure velocity is minimized. The trajectory of the space-craft is assumed to be a series of conics.

Using this approach dual flyby solutions of Jupiter and Saturn were sought for the period from approximately 1977 to approximately 2000. Most of the feasible opportunities for these missions occur for a period of several years before and after April 1981 (when Jupiter and Saturn are at the same celestial longitude) and for a few years before the next alignment in June 2000. In general, the opportunities or departure windows for these missions occur throughout the synodic period of Jupiter and Saturn at approximately 400-day intervals for EJSE opportunities and at 378-day intervals for ESJE opportunities.

An EJSE or ESJE mission has three legs, each of which can have a transfer angle of less than 180° (type I) or greater than 180° (type II). During any departure window, as many as eight possible minimum-departure-velocity solutions having a specified radius of closest approach (rca) at one of the planets may exist. The choice of a type I or type II transfer angle for the departure and return legs was easily made because these two angles are strong functions of the angular position of the most rapidly moving planet, the Earth. The choice of transfer angle for the interior leg was not made easily because this angle is between Jupiter and Saturn, which have relatively slow motions. The usual number of solutions during any departure window, therefore, is four. Usually only one, or none, can meet the acceptance criteria of low departure velocity, low entry velocity, and safe rca to the planets and to the Sun.

The minimum rca at Jupiter was fixed at 1.1 Jupiter radii (JR). The situation at Saturn was somewhat more complicated because considerable doubt exists about the lower and upper extremities of the rings of Saturn. Any solution with an rca at Saturn between 1.2 and 2.5 Saturn radii (SR) was rejected because this region is the one in which the rings of Saturn are believed to be concentrated. Solutions with an

Table 1 Characteristics of Earth-Jupiter-Saturn-Earth Trajectories (1977–1983)

	Exterior Ring Passage					Interior Ring Passage		
Departure Date	10/5/78	11/5/79	12/6/80	1/8/82	2/11/83	9/5/77	10/4/78	11/7/79
Flight Time (days)								
T_{EJ}	701.2	580.8	524.5	467.2	412.8	805.4	653.4	527.8
T_{JS}	1,060.1	979.0	1,291.0	1,563.7	1,783.9	985.1	831.4	704.2
T_{SE}	1,889.7	1,740.0	1,838.5	1,623.9	1,357.0	1,503.6	1,421.4	1,295.2
Transfer Angle (deg)								
θ_{EJ}	158.1	148.1	142.3	135.4	128.4	164.4	155.2	142.8
θ_{JS}	44.7	30.0	24.9	16.7	6.20	55.4	39.6	23.6
θ_{SE}	155.5	194.7	194.9	210.3	125.1	146.7	149.6	163.9
Semimajor Axis (au)								
a_{EJ}	3.72	4.94	6.49	10.95	92.64	3.34	4.00	6.53
a_{JS}	9.30	7.38	5.58	5.17	5.06	21.60	17.72	13.27
a_{SE}	5.52	5.76	5.73	6.21	7.54	6.16	6.62	7.94
Eccentricity								
e_{EJ}	0.731	0.799	0.848	0.910	0.989	0.699	0.750	0.848
e_{JS}	0.637	0.758	0.865	0.949	0.994	0.783	0.793	0.859
e_{SE}	0 823	0.835	0.834	0.861	0.880	0.841	0.850	0.875
C_3(km^2/sec^2)	92.8	104.0	113.0	126.6	146.7	88.9	95.6	114.6
Declination of Departure Asymptote (deg)	36.7	26.7	13.0	−30.0	−17.3	35.8	34.6	24.4
rca (planet radii)								
Jupiter	45.7	107.9	241.84	49.21	21.12	16.95	34.32	82.78
Saturn	7.1	6.75	22.0	20.88	13.0	1.188	1.108	1.02
Inclination to Equator								
(deg) of Jupiter	10.9	17.8	158.0	175.9	176.5	9.18	10.08	17.91
Saturn	149.8	158.7	159.5	154.7	152.1	150.6	150.1	148.0
Entry Velocity Magnitude (km/sec)[a]	16.211	17.736	17.216	19.466	18.283	16.268	16.014	16.698

[a] Entry velocity at 111 km perigee.

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‡ J = Jupiter; S = Saturn; U = Uranus; N = Neptune; P = Pluto; E = Earth.

Table 2 Characteristics of Earth-Saturn-Jupiter-Earth Trajectories (1979–1984)

	Exterior Ring Passage				Interior Ring Passage	
Departure Date	12/4/80	12/16/81	12/27/82	1/7/84	10/12/79	12/2/80
Flight Time (days)						
T_{ES}	1,456.9	1,498.6	1,561.7	1,606.2	1,323.7	1,298.9
T_{SJ}	2,143.5	2,151.6	2,155.3	2,145.3	1,911.1	1,946.4
T_{JE}	652.0	627.0	586.9	474.8	669.2	639.5
Transfer Angle (deg)						
θ_{ES}	157.0	158.3	160.3	162.6	194.3	153.9
θ_{SJ}	249.8	270.7	291.1	309.5	199.9	225.0
θ_{JE}	184.6	185.8	188.8	92.3	216.3	210.3
Semimajor Axis (au)						
a_{ES}	6.77	6.68	6.47	6.34	7.66	8.13
a_{SJ}	8.78	7.91	7.01	6.26	13.49	12.04
a_{JE}	3.82	4.28	5.05	6.28	3.27	3.60
Eccentricity						
e_{ES}	0.855	0.853	0.848	0.845	0.881	0.879
e_{SJ}	0.752	0.758	0.797	0.848	0.754	0.772
e_{JE}	0.749	0.782	0.824	0.869	0.744	0.766
C_3(km^2/sec^2)	129.0	125.4	120.8	115.9	241.7	131.3
Declination of Departure Asymptote (deg)	30.4	23.9	16.7	8.2	−16.2	28.2
rca (planet radii)						
Saturn	2.51	3.25	4.83	7.18	1.02	1.053
Jupiter	1.51	1.38	1.22	1.12	1.65	1.272
Inclination to Equator (deg) of Saturn	153.7	152.5	151.9	152.1	153.2	154.9
Jupiter	171.5	167.5	170.9	176.8	172.6	179.6
Entry Velocity Magnitude (km/sec)[a]	16.787	18.496	19.577	21.084	19.456	19.717

[a] Entry velocity at 111 km perigee.

rca at Saturn less than 1.02 SR were also rejected because this value probably represents the upper reaches of the Saturn atmosphere. Solutions with an rca and with a radius to the equatorial nodal point at Saturn between 1.02 and 1.2 SR were accepted; however, there were reservations because recent evidence,[2] though unconfirmed, suggests that a fourth ring around Saturn may exist in this region. Solutions with an rca at Saturn greater than 2.5 SR are referred to as exterior-ring missions; solutions with an rca at Saturn between 1.02 and 1.2 SR, as interior-ring missions.

An Earth-entry velocity of approximately 20 km/sec is considered reasonable. In most cases, if the entry velocity can be brought to an acceptable value by varying the rca at one of the planets, then the rca to the Sun on the return leg is also an acceptable value (approximately 1 au).

Results

The opportunities that were found for the EJSE and ESJE missions are given in Tables 1–4. These missions were

Table 3 Characteristics of Earth-Jupiter-Saturn-Earth Trajectories (1996–1999)

	Exterior Ring Passage				Interior Ring Passage			
Departure Date	3/29/96	4/29/96	4/26/97	5/26/98	4/15/96	4/23/97	5/26/98	7/8/99
Flight Time (days)								
T_{EJ}	1,265.0	1,255.0	807.9	617.4	1,105.7	749.6	570.9	440.7
T_{JS}	1,051.9	1,045.8	1,094.0	1,002.0	924.3	910.4	764.9	636.0
T_{SE}	1,722.3	1,753.4	1,748.5	1,677.4	1,260.5	1,250.7	1,190.0	1,071.5
Transfer Angle (deg)								
θ_{EJ}	197.1	168.8	164.5	153.8	167.6	161.4	149.5	134.5
θ_{JS}	53.9	52.6	59.0	44.5	56.9	55.5	38.1	18.7
θ_{SE}	130.0	173.5	134.3	170.7	138.6	132.0	142.3	165.4
Semimajor Axis, (au)								
a_{EJ}	3.11	3.11	3.02	3.51	3.02	3.08	3.83	8.86
a_{JS}	8.58	8.37	9.05	7.38	16.59	16.43	12.70	9.18
a_{SE}	5.01	5.10	5.01	5.16	6.28	6.29	6.91	9.38
Eccentricity								
e_{EJ}	0.679	0.687	0.668	0.711	0.676	0.674	0.735	0.885
e_{JS}	0.583	0.586	0.569	0.625	0.742	0.743	0.762	0.886
e_{SE}	0.827	0.801	0.822	0.804	0.846	0.850	0.857	0.893
C_3(km^2/sec^2)	86.2	120.1	86.2	89.1	107.4	84.6	93.8	127.5
Declination of Departure Asymptote (deg)	−40.7	−46.1	−41.1	−20.3	−46.0	−38.3	−18.9	2.3
rca (planet radii)								
Jupiter	4.55	4.38	22.6	54.0	3.43	18.5	40.3	574.7
Saturn	8.88	8.67	8.65	9.15	1.124	1.136	1.111	1.02
Inclination to Equator (deg) of Jupiter	8.76	8.69	8.63	7.62	9.42	7.98	7.81	72.45
Saturn	154.1	149.2	154.0	150.8	153.1	153.2	152.9	151.6
Entry Velocity Magnitude (km/sec)[a]	19.784	16.757	19.111	15.866	16.925	17.763	16.370	16.544

[a] Entry velocity at 111 km perigee.

Table 4 Characteristics of Earth-Saturn-Jupiter-Earth Trajectories (1997–1999)

	Exterior Ring Passage			Interior Ring Passage
Departure Date	6/16/97	6/30/98	7/12/99	6/14/97
Flight Time (days)	1,480.6	1,512.5	1,521.7	1,365.4
T_{ES}				
T_{SJ}	2,007.8	1,986.9	2,050.2	1,640.1
T_{JE}	643.9	623.6	579.5	825.9
Transfer Angle (deg)				
θ_{ES}	160.1	162.8	165.7	157.7
θ_{SJ}	176.7	192.8	216.3	144.1
θ_{JE}	133.3	106.1	108.6	233.0
Semimajor Axis (au)				
a_{ES}	5.52	5.41	5.38	5.92
a_{SJ}	10.74	10.34	8.77	21.40
a_{JE}	3.79	3.41	3.45	3.23
Eccentricity				
e_{ES}	0.817	0.812	0.811	0.829
e_{SJ}	0.602	0.633	0.627	0.782
e_{JE}	0.746	0.764	0.760	0.767
C_3(km²/sec²)	124.1	119.1	113.7	125.0
Declination of Departure Asymptote (deg)	−24.5	−15.9	−5.3	−23.6
rca (planet radii)				
Saturn	3.16	3.50	4.52	1.14
Jupiter	2.52	2.01	2.02	1.67
Inclination to Equator (deg) of Saturn	128.0	156.1	152.8	150.9
Jupiter	122.7	173.3	177.2	170.4
Entry Velocity Magnitude (km/sec)[a]	15.567	19.837	19.112	21.529

[a] Entry velocity at 111 km perigee.

chosen because they satisfied, more or less, the arbitrary acceptability criteria of low departure velocity, low entry velocity, and safe rca to the planets and to the Sun. A continuum of missions exists in the vicinity of each of those given in the tables. The characteristics of each mission can be changed by applying a different set to acceptability criteria.

Opportunities for the EJSE missions will be present in each year from 1977 to 1983 (Table 1) and again from 1996 to 1999 (Table 3). Several opportunities will be present during the years from 1984 to 1996, but most of these opportunities will result in collisions with Jupiter and in extremely high entry velocities, or will come too close to the Sun on the interior or return legs. All these opportunities have entry velocities less than 20 km/sec, and all but one opportunity (February 1983) have energies (C_3) of less than 130 km²/sec².

Opportunities for the ESJE missions will be present each year from 1979 to 1984 (Table 2) and again from 1997 to 1999 (Table 4). Several opportunities will be present during the periods of 1985 to 1996; but, as in the EJSE opportunities, these opportunities will violate the mission-acceptance criteria—most seriously by collisions with Jupiter. All these opportunities except the interior-ring mission of 1979 will have energies (C_3) of about 130 km²/sec² or less. All these opportunities except the exterior-ring mission of 1984 and the interior-ring mission of 1997 will have entry velocities of less than 20 km/sec. The ESJE exterior-ring opportunity of December 16, 1981, is shown in Fig. 1.

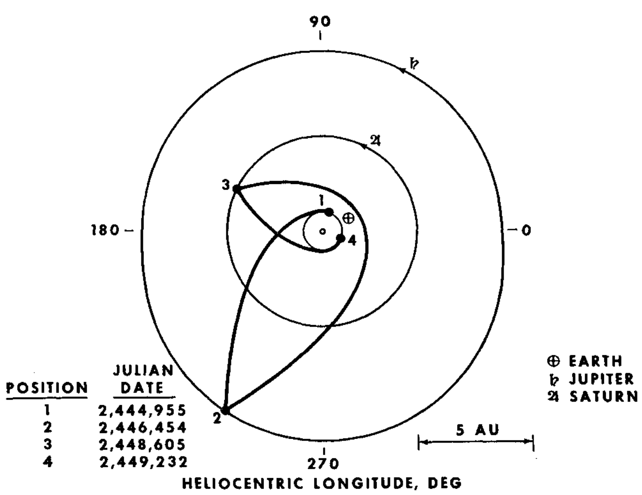

Fig. 1 **December 1981 ESJE opportunity.**

References

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